Practice

Use algebraic reasoning to solve the following equations for the given variable. Show all work.
Use your calculator ONLY for equations involving *e* and *natural log*.
For the trigonometric equations, find solutions $0 \le \theta \le 2\pi$

1. $e^{2t} - 26e^{t} + 25 = 0$

$(e^t - 25)(e^t - 1) = 0$

$e^t - 25 = 0$ | $e^t - 1 = 0$

$e^t = 25$ | $e^t = 1$

$t = \ln 25$ | $t = \ln 1$

2. $2(\ln x)^2 + \ln x - 1 = 0$

$m = \ln x$ $\quad 2m^2 + m - 1 = 0$

$(2m - 1)(m + 1) = 0$

$2m - 1 = 0$ | $m + 1 = 0$

$m = \frac{1}{2}$ | $m = -1$

$\ln x = \frac{1}{2}$ | $\ln x = -1$

$x = e^{\frac{1}{2}} = \sqrt{e}$ $\quad x = e^{-1} = \frac{1}{e}$

3. $y^{10} - 5y^6 + 4y^2 = 0$

$y^2(y^8 - 5y^4 + 4) = 0$

$y^2(y^4 - 4)(y^4 - 1) = 0$

$y^2 = 0$ | $y^4 - 4 = 0$ | $y^4 - 1 = 0$

$y = 0$ | $y = \sqrt[4]{4}$ | $y = \sqrt[4]{1}$

$y = \pm\sqrt[4]{4}$ | $y = \pm 1$

4. $x^6 + 9x^3 = 2x^3 + 8$

$x^6 + 7x^3 - 8 = 0$

$(x^3 + 8)(x^3 - 1) = 0$

$x^3 = -8$ | $x^3 - 1 = 0$

$x = \sqrt[3]{-8}$ | $x = \sqrt[3]{1}$

$x = -2$ | $x = 1$

5. $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ | $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ | $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ | $x = \pi$

6. $3(x+2)^2 - 7(x+2) + 4 = 0$

$m = x + 2$ $\quad 3m^2 - 7m + 4 = 0$

$(3m - 4)(m - 1) = 0$

$3m - 4 = 0$ | $m - 1 = 0$

$m = \frac{4}{3}$ | $m = 1$

$x + 2 = \frac{4}{3}$ | $x + 2 = 1$

$x = -\frac{2}{3}$ | $x = -1$

7. $3x^6 - 20x^3 = 32$

$3x^6 - 20x^3 - 32 = 0$

$(3x^3 + 4)(x^3 - 8) = 0$

$3x^3 + 4 = 0$ | $x^3 - 8 = 0$

$x = \sqrt[3]{-\frac{4}{3}}$ | $x = \sqrt[3]{8}$

$x = 2$

8. $(\log_2 x)^2 - 10\log_2 x = -16$

$m = \log_2 x \quad m^2 - 10m = -16$

$m^2 - 10m + 16 = 0$

$(m - 8)(m - 2) = 0$

$m - 8 = 0$ | $m - 2 = 0$

$m = 8$ | $m = 2$

$\log_2 x = 8$ | $\log_2 x = 2$

$x = 256$ | $x = 4$

9. $2t^4 + 3t^2 = 2$

$2t^4 + 3t^2 - 2 = 0$

$(2t^2 - 1)(t^2 + 2) = 0$

$2t^2 - 1 = 0$ | $t^2 + 2 = 0$

$t = \sqrt{\frac{1}{2}}$ | No solution

10. $4\sin^2 x = 4\sin x - 1$

$4\sin^2 x - 4\sin x + 1 = 0$

$(2\sin x - 1)(2\sin x - 1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

11. $(8n - 7)^2 = 7 - 8n$

$(8n - 7)^2 = -(8n - 7)$

$(8n - 7)^2 + (8n - 7) = 0$

$(8n - 7)((8n - 7) + 1) = 0$

$8n - 7 = 0$ | $8n - 6 = 0$

$n = \frac{7}{8}$ | $n = \frac{3}{4}$

12. $\frac{1}{(x-1)^2} - \frac{1}{(x-1)} - 2 = 0$ $\quad (x-1)^2$

$1 - (x - 1) - 2(x - 1)^2 = 0$

$2(x - 1)^2 + (x - 1) - 1 = 0$

$(2(x - 1) - 1)((x - 1) + 1) = 0$

$2(x - 1) - 1 = 0$ | $(x - 1) + 1 = 0$

$2x - 2 - 1 = 0$ | $x = 0$

$2x = 3$

$x = \frac{3}{2}$